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Estimating the Cyclical Component from Annual Time Series

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Analytical Services Branch

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CONTENTS

ESTIMATING THE CYCLICAL COMPONENT FROM ANNUAL TIME SERIES: A CASE STUDY WITH AUSTRALIAN MULTIFACTOR PRODUCTIVITY

Mark Zhang and Lewis Conn
Methodology Division

ABSTRACT

When analysing an economic data series, we look for underlying patterns to isolate areas of interest and exclude elements that are not closely related to our interests. The ABS currently uses an 11-term Henderson filter as the detrending method to separate out the long-term trend and cyclical components within Australian multifactor productivity (MFP). This paper investigates what are the appropriate statistical methods to extract the cyclical component for annual time series using desirable frequency properties and model fitness in the time domain. As a case study, rather than using structural economic modelling techniques, we apply several commonly used filters to the annual MFP series and evaluate their performance. We conclude that care has to be taken in order to use different methods in relation to their frequency properties, and the volatility level of the series under study.

As result of this investigation and with considerations of stability, revisions and comparability, the customised Hodrick–Prescott filter with a smoothing parameter of 25 appears to perform better than other methods in extracting the MFP cycle. Nonetheless, there remain several statistical challenges to overcome. The methodology presented in this paper paves the way to formalise a general approach to cycle extraction which can be extended to other ABS annual time series.

1. INTRODUCTION

When analysing an economic data series, we look for underlying patterns to isolate areas of interest and exclude elements that are not relevant to our interests. These underlying patterns can include long term trends, seasonal effects, cycles and irregular components. With monthly and quarterly economic data, the ABS generally publishes the original, seasonally adjusted, and trend estimates separately to allow users to make informed decisions. Trend estimates consist of (long-term) trend and cycle components but the cycle component is not usually estimated separately or published by the Australian Bureau of Statistics (ABS).

The ABS compiles annual estimates of the multifactor productivity (MFP) index for the Australian market sector, which are published in the *Australian System of National Accounts* (ABS cat. no. 5204.0). As an index, MFP reflects the combined effects of technological, efficiency, capacity utilisation and resource allocation changes by measuring the ratio between the outputs and inputs over time. In the Australian System of National Accounts, outputs are measured as the volume of value added and inputs are measured by labour and capital.

The ABS currently publishes the MFP growth between adjacent years as well as average growth rate between MFP cycle peaks to minimise the influence of cyclical effects on measures of capital and productivity. This peak-to-peak adjustment method approximates, on average, a close normal capacity utilisation (Morrison, 1985).

> "Macroeconomists have become interested in the cyclical behaviour of productivity because of the realization that procyclicality is closely related to the impulses or propagation mechanisms underlying business cycles. Even the cyclical mismeasurement that was formerly dismissed as unimportant turns out to be a potentially important propagation mechanism." (Basu and Fernald, 2001).

The ABS currently uses an 11-term Henderson filter as a detrending method. However, this method has not previously been subjected to detailed methodological study in order to understand and assess its suitability relative to alternatives. The ABS has undertaken a project to better understand the cycles of annual MFP. As part of the project, this paper investigates various statistical methods used to extract the cyclical component from annual time series based on their desirable frequency properties and model fitness in the time domain. As a case study, rather than using structural economic modelling techniques, we study the effect of several commonly used filters on the annual MFP series and evaluate their performance.

This investigation found that the cycle components for MFP were reasonably consistent across the various filters. With considerations of stability, revisions and comparability, the customised Hodrick–Prescott filter appears to perform better than other methods in extracting the MFP cycle. Nonetheless, several statistical challenges remain to be overcome.

This paper is structured as follows. The main part presents major discussions and results which are supported by detailed technical materials in the Appendixes. Section 2 briefly reviews different statistical approaches for cyclical component extraction. Section 3 describes two major categories of trend extraction methods and defines the different types of filters used in the study. Section 4 presents a case study for estimating the cyclical component from the annual MFP time series at the aggregate market level. From this case study, we demonstrate that care needs to be taken when the noise level of a series is high, and suggest a way to deal with this situation as well as discuss revisions issues. In Section 5, we present some applications of the methodology to other economic series and indutry level MFP cycle analysis. Section 6 summarises our findings, presents our recommendations and remarks. It argues that there is a strong case for carrying out MFP cycle extraction using a customised HP filter to take care of the level of noise.

2. COMPONENTS OF A NON-STATIONARY ANNUAL TIME SERIES

A non-stationary annual time series y_t can be considered as having a long-term trend (u_t) component, and a stationary (e_t) component as depicted by equation (1) [1]

$$y_t = \mu_t + e_t\,, \quad t = 1, \ldots, T \tag{1}$$

where t denotes year.

It is necessary to define precisely what is meant by a cycle in the classical sense. Cyclical behaviour is the subject of much economic research, as well as of much debate. [2] In the literature, the cyclical component of Gross Domestic Product (GDP) is often referred to as the business cycle of the economy. Therefore, some of the statistical methods used for the estimation of the cyclical component of GDP are often applied to other macro-economic time series in the study of business cycles. [3]

There is no agreement on the causes and nature of business cycles. For an overview of current opinions, we refer to some of the extensive literature: Zarnowitz (1987), Prescott (1986), Cooper (1997), Fuhrer and Schuh (1998). Given this lack of unambiguous theoretical foundations, it is not surprising that measuring the state of an economy is a much debated issue. Partly, it is down to how one defines business cycles. Most economists still agree on the description given by Burns and Mitchell in 1946:

> "Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organise their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years."

Cyclical behaviours are generally observed as recurrent phenomena with typical frequencies and the existence of a cycle can be defined as a peak in the spectrum of a time series in a specified range. It seems natural to concentrate on the spectral mass contained in specified frequency ranges, and we therefore propose the following:

1 This is a simplified decomposition form. Other forms of decomposition future decompose the stationary part into a smoothing cyclical and irregular components.

2 The debate also includes whether a cyclical component should be smooth or not. Both smoothing cyclical component and non-smoothing (smoothing cyclical and irregular) component can be referred as cyclical component depending on the context of estimation methods/models in this paper.

3 In the literature some papers refer to the cyclical component of any economic indicator time series as the business cycle component of the economic indicator.

Definition. An economic time series exhibits a classical cycle if there is significantly more spectral mass in the cyclical component range of 2 to 10 years.

Based on this definition for the cyclical component, we can define the long term trend (referred as trend hereafter) as the variations in a time series with frequencies longer than ten years and the irregular component as the variations with frequencies less than two years.

Just as there is no agreement on the causes or nature of business cycles, there is also much debate on how to measure these phenomena. There are basically two different concepts, and no matter which one is chosen , the choice will be open to criticism.

- **Deviations from trend (cyclical) approach**: this method defines the cycle as the deviation of a time series from its long-term trend. The concept of potential growth is well-founded in economic theory, and determining whether an indicator is developing above or below the trend relays important information. Therefore, the deviation from trend approach enables a more thorough characterisation of the dynamics of short-term economic indicators. This method is often used to estimate the output gap.
- **Growth rate cycle approach**: this method involves a study of the cyclical behaviour of the growth rate of an indicator time series. As most economic time series are rather volatile, it is necessary to filter the growth rates to separate the important developments from noise.

Using the deviations from trend approach, we can easily compare and combine the development of different indicators such as industry level MFP, as they are all translated to standardised cycles. This approach highlights the dynamics most relevant for cyclical component analysis. The estimated cyclical components are in general easier to interpret than the noisy unfiltered realisations. This approach offers a ready and clear framework to classify the state of the industry level MFP. Because of these considerations, we decided to apply our analysis using the cyclical approach.

3. CYCLICAL COMPONENT EXTRACTION METHODS

Under the cyclical approach, the most serious obstacle to extracting the cyclical component is that economic time series contain fluctuations as well as the trend component, and that the nature of the interaction between cyclical and trend components is not sufficiently understood. The difficulty arises since neither the trend nor the cycle is directly observable. In general, the different approaches to estimating trends are classified into: (1) statistical detrending and (2) estimation of the structural economic relationship. The difference is that the former approach attempts to separate the time series into permanent and cyclical components while the latter isolates the effects of structural and cyclical influences using economic theory. We will focus on statistical detrending methods in this paper.

A number of statistical techniques for estimating the trend and cyclical components have been developed in the literature. However, many researchers believe that none is completely satisfactory. This lack is manifested through the many empirical studies showing that different methodologies and assumptions for estimating trend and cyclical components can produce different results. There are two major categories of statistical trend extraction methods.

1. The first category can be classified as filtering methods which extract trends using low pass filters, leaving just the stationary part of the series. These filters are based on the desirable characteristics of filter frequency response functions, and applying their implied weighting patterns to the time series under study. The Hodrick–Prescott filter (Hodrick and Prescott, 1997) and Baxter–King filter (Baxter and King, 1999) are common examples of such filters used in economic time series applications. The discussion in Canova (1998a, 1998b) and Burnside (1998) makes clear that different detrending methods emphasize particular frequency ranges in the data, and that many stylized facts are sensitive to the choice of detrending method or trend estimator. However, the implied model of a specific filter may not necessarily fit the time series in time domain.

2. The second category is model based signal extraction. This method fits a specific model to the data first to approximate the data generation process of the time series under study, and then to provide a set of implied weighting patterns which extract the trend. The unobserved component model (Harvey 1989) and the Beveridge–Nelson model (Beveridge and Nelson, 1981) are such examples. Model-based signal extraction is very sensitive to the appropriateness of the model specification used to describe the data generation process. This dependence would produce unexpected result when the model is mis-specified.

Although the two methods focus on different perspectives, they are closely linked and can have equivalent presentations of each other. (Some examples can be found in Appendix B.) The link is the signal–noise ratio (or smoothing parameter which is the inverse of signal–noise ratio). The signal can be defined as the variance of the non-stationary trend component, and the noise can be defined as the variance of the stationary cyclical and irregular components. The signal–noise ratio plays a key role in determining how observations should be weighted for signal extraction. A higher signal–noise ratio means that the closest observations receive more weight.

In frequency domain arguments, the signal–noise ratio translates into a predetermined cut-off period (or frequency) to extract the trend. Any predefined filter approach could produce a spurious cycle. For example, when Cogley and Nason (1995) applied the Hodrick–Prescott filter to a random walk series, a large and persistent cycle emerged. In time domain arguments, the signal–noise ratio would be different for series with different characteristics. Therefore, a univariate model based on signal extraction would produce two very different trends in a situation of two cointegrated series exhibiting balanced growth with different level of volatility. For example, investment and GDP are usually assumed to have a common trend, but it is an established stylised fact that the volatility of investment is greater than that of GDP. (See the theoretical argument and an empirical simulation result in Appendix C.)

Much work in the area of estimating the cyclical component has focussed on the U.S. quarterly GDP series as a part of business cycle study. Hodrick and Prescott (1997) suggested that a signal–noise ratio set to 1/1600 was optimal to extract the trend component and this has been taken as a universal constant for economic time series. There has been a good deal of debate and written work on how this parameter should be adapted to series other than U.S. GDP and over different observation intervals. We list some work on the annual time series. Backus and Kehoe (1992) used a signal–noise ratio of 1/100. Maravall and del Ria (2001) suggest 1/6 and 1/7. More recently, Ravan and Uhlig (2002) analysed the issues and suggest 1/6.25. Harvey and Trimbur (2003) take a similar view, but focus more on the implication to the unobserved component model and agree that 1/6.25 for annual data series matches 1/1600 for quarterly data series.

3.1 Discussion of filters

In our investigation, we consider the following detrending methods commonly used in the macro-economic time series cyclical component analysis.

1. 11-term Henderson (Henderson, 1916) filter: A linear two-sided low-pass filter which is used in the X–11 seasonal adjustment method family (U.S. Census X–11, Statistics Canada X–11–ARIMA, and U.S. Census X–12–ARIMA) to produce a

combined trend and cycle component. This filter is currently used by the ABS with Australian market MFP and will be referred to as H(11) hereafter.

2. Hodrick–Prescott (Hodrick and Prescott, 1980) filter: A linear two-sided low-pass filter which extracts a trend component by introducing a trade off between a good fit to the actual series and the degree of smoothness of the trend series. As discussed above, this smoothing restriction parameter is often set to 6.25 in the economic literature based on U.S. annual GDP (translating to a signal–noise ratio of 1/6.25). This filter will be referred to as HP(6.25) hereafter.

3. Baxter–King (Baxter and King, 1995) filter: A two sided band pass filter which eliminates very slow moving ('trend') components and very high frequency ('irregular') components while retaining the intermediate ('cycle') components. Our study uses a filter length of 17 and the lower and upper boundaries set as 2 and 10 years respectively. This filter is referred to as BK(2, 10, 17) hereafter.

4. Beveridge–Nelson (Beveridge and Nelson, 1981) method: This model-based method assumes an ARIMA(p,1,q) model fits the time series and imposes restrictions to decompose the trend and cycle components. As the trend is a stochastic process, this model may not necessarily produce a smooth trend. For our analysis, an ARIMA(2,1,2) is used, referred to as BN(2,2). The Beveridge–Nelson approach implies much of the variation of the series is attributable to variation in the trend while the cycle component is small and noisy (Morley *et al.*, 2003).

5. Unobserved components models (Harvey, 1993): This model-based method assumes that a time series is composed of trend, cycle and irregular components. These three components can be recovered by imposing certain restrictions on the trend and cycle process. This model is referred as the (structural state space) UCM hereafter. Utilising Kalman filter techniques, we can produce a 'smooth trend' from the UCM. (See details in Appendix B.)

Some statistical techniques often featured in the literature were not selected for practical reasons. These include non-linear trend methods, first order differences, Markov-switching models (Hamilton, 1989), exponential smoothing (Gardner, 1985) and the Rotemberg decomposition (Rotemberg, 1999). Others, like the non-parametric method of the Phase Average Trend (Boschan and Ebanks, 1978) were not considered because they are designed for quarterly and/or monthly time series.

Harvey and Jaeger (1993) and Harvey and Trimbur (2003) show that both Hodrick–Prescott and Baxter–King filters are special forms of the Butterworth filter family and are partially equivalent to the unobserved component model framework by explicitly modelling trend and cycle. However, a general unobserved component model does not suffer the criticism of producing spurious cycles as far as we know.

Although a general unobserved component model is preferable from a theoretical point of view, it is based on different principles from filtering methods and is not easy to implement in practice because of cumbersome model fitting and potential model instability. (See details in Appendix B.)

It is difficult to identify a key set of diagnostics to determine the optimal filter and extract the cyclical component for a particular economic data series. However, if there is a strong cyclical component in the data, it will dominate the effects of the detrending filter, and the results will be roughly equivalent for different methods.

In answering two fundamental questions –

(1) What is the appropriate smoothing parameter of the Hodrick–Prescott filter for the annual Australian MFP time series?

(2) Whether the cyclical component derived from the HP filter is spurious?

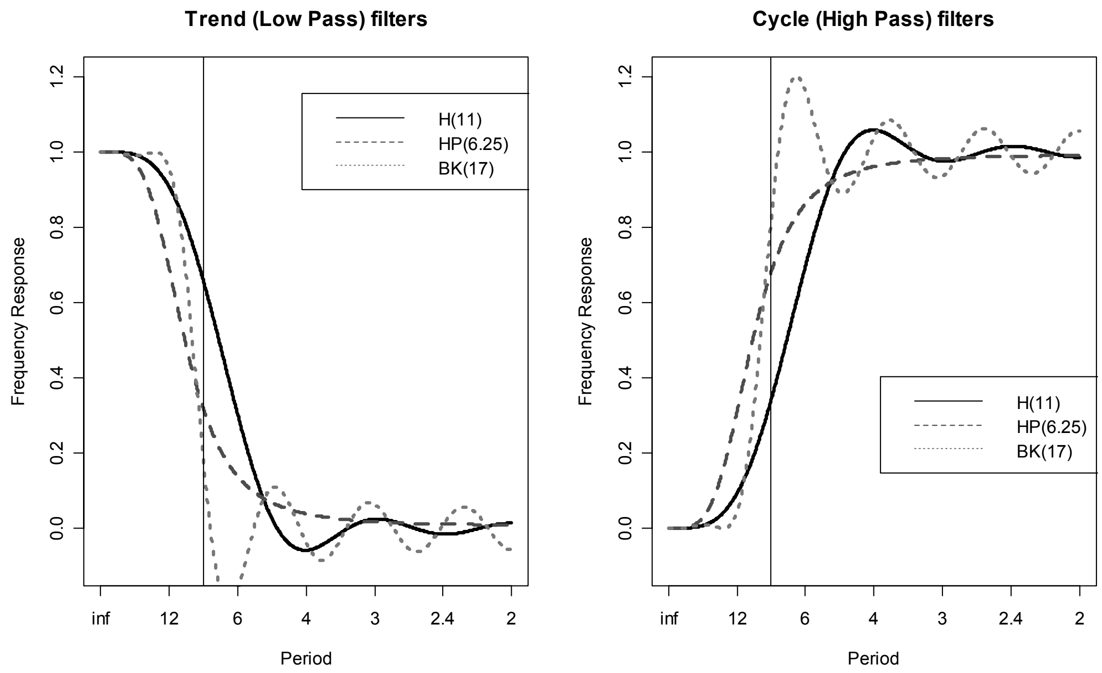
we use the UCM as a model-based companion to verify the potential risk of spurious cycles and to estimate the signal–noise ratio (as an estimator of the HP smoothing parameter) which offers a rough idea of plausible values of the way in which these values might change with different levels of volatility.

3.2 Frequency properties of filters

The frequency properties of a filter can be presented as a gain function, which shows how the spectrum power of a time series is suppressed as the result of applying a filter to the series. Figure 3.1 shows the gain functions of low-pass and high-pass filters (the gain function of a high-pass filter is equal to one minus the that of the low-pass filter) derived from the Henderson filter H(11), the Hodrick–Prescott filter HP(6.25) and the Baxter–King filter BK(2,10,17) for large samples and t not near the beginning or end of the series. The vertical line is a 10-year period reference line.

The high-pass filter in figure 3.1 shows that the H(11) filter suppresses more power at low frequencies than the HP(6.25) filter and BK(2,10,17) filter. Both the H(11) and BK(2,10,17) filters have an oscillating nature at a period of less than eight years, which may lead to an increased chance of those methods producing spurious cycles.

3.1 Frequency response functions



We use the one-half point of a filter as the point at which it has a gain of one-half to indicate its filtering property.[4] The spectrum power is mostly retained above this point and is largely suppressed below this point. Hence the one-half point provides good indication of the cut-off point of a filter.

3.2 One-half point

	Henderson H(11)	*Hodrick–Prescott HP(6.25)*	*Baxter–King BK(2, 7, 10)*
Period (years)	6.94	9.80	8.95

Based on our definition that a cyclical component should be in the range of 2 to 10 years, the one-half point of the Henderson seems too low, with cycles greater than 7 years being largely suppressed by the filter. This indicates that the H(11) trend estimate absorbs too much cycle information at the low frequency range. That is, the lower frequency cycles are treated as part of the trend by H(11). In addition, the gain function of H(11) shows that it amplifies cycles with periods in the range from four to six years. The BK(2,10,17) has multiple areas of amplified cycles. In other words, both the Henderson and Baxter–King filter are likely to product spurious cycle between ranges with amplified cycles.

4 One-half point of a filter gain function is a crude indictor of the cut-point of the filter. It is not sufficient by itself to describe the full frequency response of the filter.

From the frequency argument point of view, the HP(6.25) is more appropriate than H(11) or BK(2,10,17) as it does not have amplified cycles and its one-half point lines up closely with our definition of a maximum cycle length of 10 years. However, this frequency argument is not sufficient for the extraction of a good quality long term trend under high levels of volatility. (See details in Appendix C.) The frequency argument should be used as a lower boundary for the choice of the HP filter smoothing parameter. The smoothing parameter will need to be adjusted if the trend variations are much lower than the level of noise (i.e. low signal–noise ratio). In the next section, we use annual MFP as a case study to choose an appropriate smoothing parameter. We also consider the GDP and labour input (Hours Worked index) series for cross-validation of the techniques.

4. CASE STUDY – AUSTRALIAN MULTIFACTOR PRODUCTIVITY CYCLES

Multifactor productivity (MFP) is one of the key drivers of economic growth and is generally linked to increasing living standards in the long run through higher real incomes. Productivity statistics are important for policy makers, researchers and economic commentators who are interested in economic growth. The Australian Bureau of Statistics (ABS) produces annual estimates of the multifactor productivity (MFP) index for the Australian market sector as part of the *Australian System of National Accounts* (ABS cat. no. 5204.0). Estimates are produced from the financial year 1964–65 allowing analysts to measure the growth of MFP over time. The ABS currently publishes MFP growth using two methods:

1. Between adjacent years;
2. Average growth rate between MFP cycle peaks.

The latter is a more consistent indicator for comparing MFP over time to minimise the influence of cyclical effects on measures of capital and productivity. This peak-to-peak adjustment method approximates, on average, a close normal capacity utilisation (Morrison, 1985).

We undertook this project to investigate statistical methods used to extract the cyclical component of MFP, and therefore identify and date MFP peaks with the aim of evaluating the methods and assumptions used. This work also examines the relationship of MFP estimates at the industry level, comparing individual industry cycles to the aggregate productivity cycle.

In the context of analysing Australian macro economic time series, the HP filter technique has been widely used in estimating total factor productivity trend in output gap estimates (Brouwer, 1998) and for output gap estimates (Gruen *et al.*, 2002) although authors of both papers acknowledge that the choice of the HP filter smoothing parameter is arbitrary.

4.1 Testing the MFP series trend and cycle components

To determine whether the MFP series has a non-stationary trend and a stationary cycle component, we can use a variety of statistical tests to help us to understand the nature of the series before we interpret the result of the trend and cyclical component estimated from a filter or model.

1. *Test if the series is non-stationary with a deterministic time trend.* Nelson and Plosser (1982) used a unit root test to show that most economic time series could not be handled by a deterministic time trend plus a stationary component. The deterministic trend and stationary models are so implausible that they should not be imposed unless there is very strong supporting evidence for doing

so. A test of the null hypothesis that the series has a unit root with a deterministic time trend can be performed by using the Dickey–Fuller test (Dickey and Fuller, 1981).

2. *Test if the series is a random walk.* Cogley and Nason (1995) show that the Hodrick–Prescott filter can produce a spurious cyclical component when it is applied to a random walk that has no cycle component. A random walk test can be performed by applying white noise tests, such as Box–Ljung Q-statistics and Durbin–Watson tests, on the differenced time series.

3. *Test the stochastic cyclical component and estimate the signal–noise ratio.* Assuming that an UCM with state space presentation is adequate to fit the time series data generation process, we can identify the nature of the trend. We can also test if a stochastic cyclical component exists, and estimate the signal–noise ratio as an assessment for what might be an appropriate smoothing parameter to use when applying the Hodrick–Prescott filter. (Details of our model selection procedure can be found in Appendix B.)

Table 4.1 shows test results for MFP, GDP and labour input series.

4.1 Tests of Multifactor productivity, Gross domestic product and Labour input properties

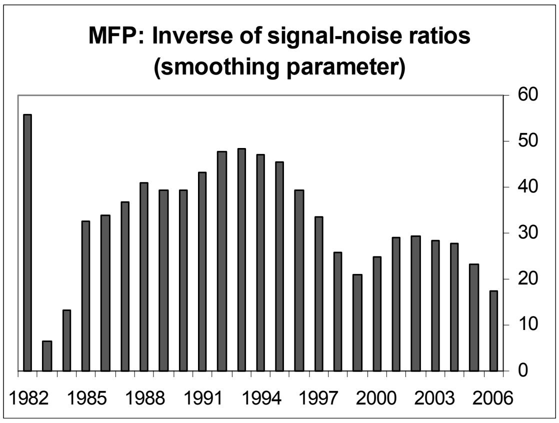
	Multi-factor productivity	*Gross domestic product*	*Labour inputs*
I(1) with deterministic trend			
Dickey–Fuller unit root tests	not rejected	not rejected	not rejected
rho p-value	0.2100	0.4483	0.0095
tau p-value	0.3200	0.2929	0.0506
Random walk			
Box–Ljung Q(6) p-value	0.3364	0.5532	0.0822
Durbin–Watson p-value	0.2418	0.0015	0.3418
UCM does not include stochastic cycle			
Inverse of signal–noise ratio (lambda)	16.7818	0.6418	0.0162
UCM includes stochastic cycle			
Inverse of signal–noise ratio (lambda)	2.98E+12	2,840	4.36E+08
Final UCM model			
Inclusion of stochastic cycle component	no	no	no

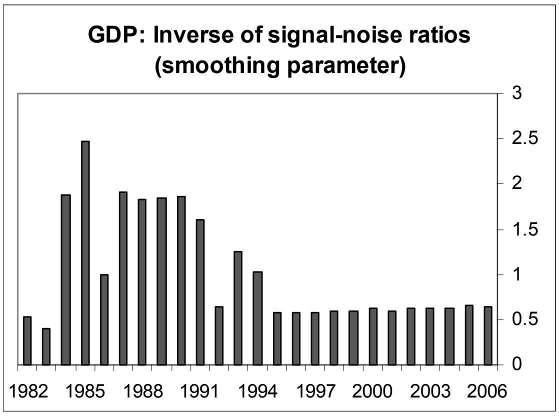
The following discussion focuses on the MFP series. The Dickey–Fuller unit root test suggests that the MFP series is not a deterministic time trend plus a stationary series because the residuals from a deterministic trend are not stationary. Both the Box–Ljung Q-statistics and Durbin–Watson test suggests that we have no strong statistical evidence to reject the hypothesis that MFP is a random walk. The UCM model test results with and without a stochastic cycle also show that there is no strong evidence of the existence of a stationary cyclical component (see details of our model selection procedure in Appendix B) if we believe the UCM fits the MFP data generating process well. The large inverse signal–noise ratio value from the UCM with a stochastic cyclical component suggests that MFP has a deterministic trend with a non-stationary stochastic cyclical component or no cyclical component at all.

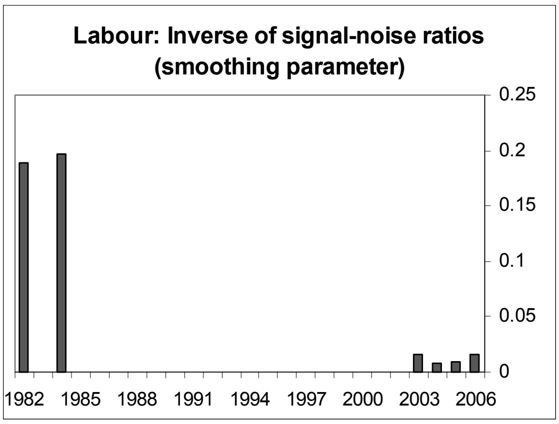
However, the inverse of the signal–noise ratio from the UCM without a cyclical component model suggests that there is more noise in the MFP series. The 'standard' HP filter with 6.25 smoothing parameter is designed for US GDP and may not be appropriate for Australian MFP, which exhibits greater volatility. Tests for GDP and labour input series have similar results, except that the Dickey–Fuller unit root test suggests that the labour input series is likely to have a deterministic time trend and both GDP and labour input series have inverse signal–noise ratios smaller than 6.25.

To determine the appropriate HP filter smoothing parameter for MFP detrending, we use a smooth trend UCM to estimate the inverse signal–noise ratios for each year starting from 1982 to 2006 (note: the UCM can estimate a deterministic trend rather than a smooth trend if a series is too short). Figure 4.2 shows the estimated inverse signal–noise ratios for MFP, GDP and labour input at each year.

4.2 Inverse signal–noise ratios of unobserved components model







The variations in the estimated signal–noise ratios show that the ratio is sensitive to data and its length. A fixed value would be preferred although the HP filter would produce similar detrending results if different smoothing parameters (or inverse signal–noise ratio) within a sensible range were used. The UCM estimated inverse of signal–noise ratios for GDP and labour input series are much smaller than 6.25 over the same periods. Therefore, use of a fixed smoothing parameter HP(6.25) is appropriate from the cycle frequency argument perspective for GDP and labour input. MFP shows consistently higher smoothing parameters reflecting the additional volatility in the series. It is questionable to use the HP(6.25) based on the frequency argument alone.

4.2 Comparison of trend filters

From the frequency argument point of view, a suitable filter should be able to separate the long-term trend and stationary components (including the cyclical component) at a particular cut-off period (e.g. ten years). However, a detrending filter will also cut out the low frequency power of the stationary component. In other words, the low frequency cycle overflows into the trend frequency range and is removed. Therefore, a detrending filter can potentially distort the properties of the detrended series. (See details in Appendix C.) Care needs to be taken when using a larger smoothing parameter derived from the signal–noise ratio from an appropriate UCM model to ensure the quality of detrending.

In choosing an appropriate fixed smoothing parameter (i.e. inverse signal–noise ratio) for MFP, we consider the following factors:

- Volatility: An appropriate fixed smoothing parameter is needed to handle the high volatility (or noise) level of the MFP series.
- Comparability: When analysing industry level MFP series and contributions to total market MFP we require the same smoothing parameter. This allows cross sectional industry level MFP cyclical comparisons and the contribution to the total MFP cyclical pattern to be meaningful and coherent. Using different customised smoothing parameters for different industry MFP series will make comparability impossible.
- Consistency and transparency: It is also desirable to use the same smoothing parameter over time consistently, and to allow for the reproduction of results.

As a result of the above considerations, we chose 25 as an appropriate smoothing parameter based on our judgement and empirical the evidence from our analysis of the signal–noise ratio and its variations over time using UCM fittings in the previous sections.

The following table shows the one-half points of HP(0.6), HP(16.78) and HP(25) against HP(6.25) and other filters. (More detailed discussions about frequency arguments can be found in Appendix A.)

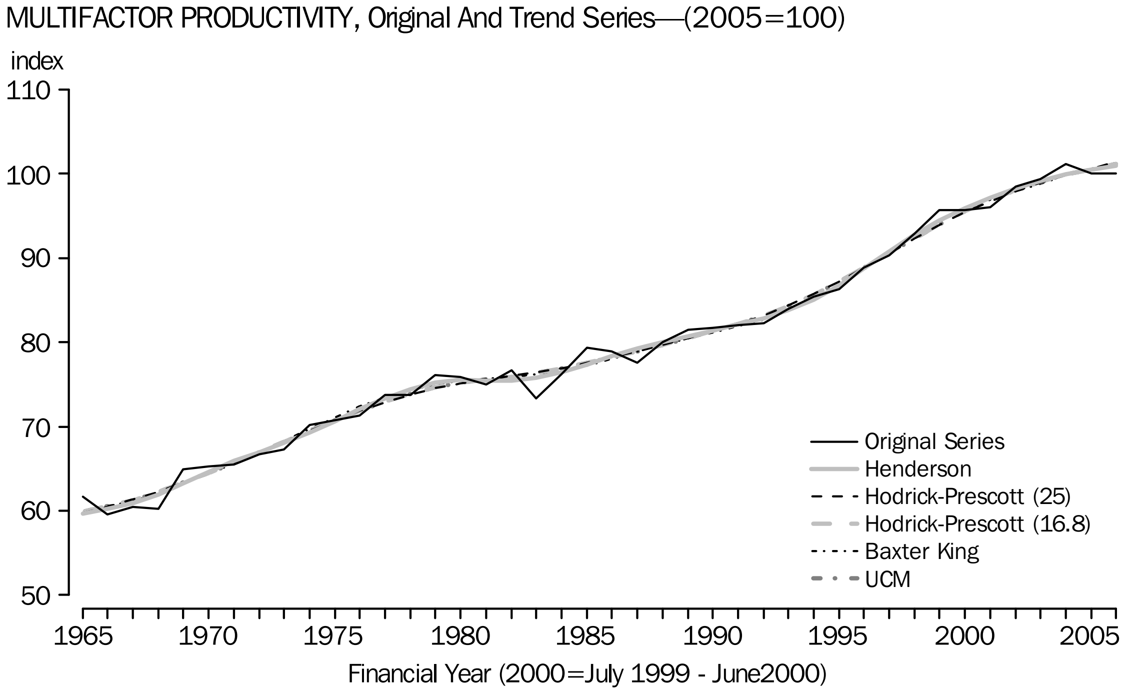
4.3 One-half point comparisons of different filters

	Henderson	*Hodrick–Prescott*				Baxter–King
	H(11)	*HP(0.60)*	*HP(6.25)*	*HP(16.78)*	*HP(25)*	*BK(2, 7, 10)*
Period (years)	6.94	5.19	9.80	12.57	13.94	8.95

The one-half point of H(25) of around 14 years is much longer than the maximum defined cycle length of 10 years. However, this adjustment is justified, given the higher level of volatility associated with MFP. (See our argument and empirical simulation study in Appendix C.) This result also suggests that the 11-term Henderson filter, H(11), may not be the most appropriate choice for the relatively high volatility MFP series.

Figure 4.4 shows the long term trends estimated from the various methods proposed.

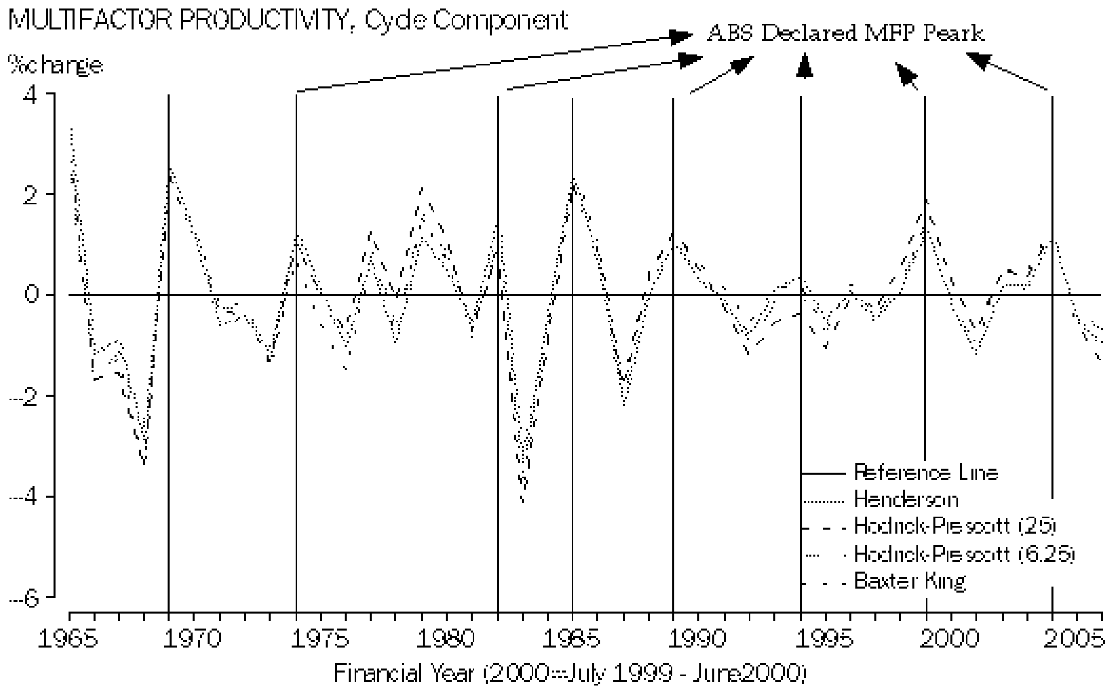
4.4 Original and trend series



Source: ABS Cat No. 5204.0

Figure 4.4 shows that MFP is a non-stationary series that is steadily increasing over time and its long term trend is captured by all methods. The differences between the estimated long term trends are reasonably small and cannot be analysed from this graph. Analysing the derived cycle component as the percentage deviation of the original series from the long term trend shown in figure 4.5 shows that the derived corresponding cyclical components do have visible differences.

4.5 Cycle component with ABS declared productivity peaks



The pattern of cyclical components from the different methods in Figure 4.5 is consistent, with almost all peaks and troughs lining up. However, in comparing the cyclical peaks from the various methods to the ABS declared peaks indicated by the vertical lines, there appear to be some missed peaks in the late 1970s and inconsistent results with the 1993–94 peak. Although identified statistical peaks provide useful evidence, the ABS considers a range of other economic information including real output (GDP), unemployment rate and business expectations when identifying productivity peaks.

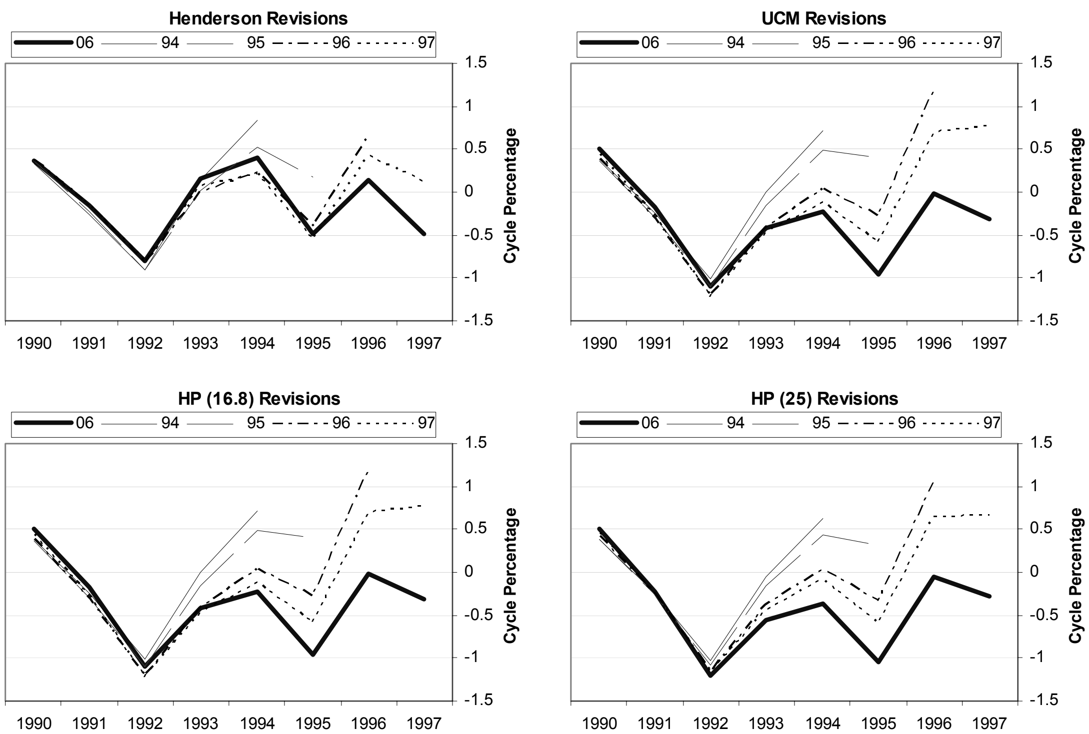
The 1993–94 declared peak is not strong under 11-term Henderson method and does not appear as a peak under the HP and unobserved components models. Given these results and the issues of the Henderson filter discussed in Section 3.2, this leads us to question whether the Henderson filter appears to be producing a spurious peak at the 1993–94. The HP filter methods produce consistent results with smoothing parameters of 25 and 16.8 and are also consistent with the unobserved components model. The Baxter–King filter produces results consistent with the HP filters, but cannot produce estimates at the end of the series (e.g. with 2005–06 data, it can only produce a trend up to 2004–05), which does not meet our timeliness criteria.

4.3 Sensitivity of methods to revisions

Many trend-cycle decomposition methods suffer from the so called end-point problem. The trend estimate at t is based on information available up to and including period t. It can change significantly when new data for period $t+1$ becomes available. Near the end of the time series sample, less information is available regarding the persistence of shocks, rendering the decomposition of the trend-cycle less reliable.

Our revision analysis was undertaken to simulate real time estimates. As the major difference between the methods was at the controversial MFP peak at 1993–94, we simulated four consecutive cyclical component estimates for 1993–94 and the last estimate made with data up to 2005–06. Figure 4.6 shows the revision patterns of the different methods.

4.6 Cycle component revision at the 1994 peak



From figure 4.6, we can observe that the 11-term Henderson filter, H(11), generally produces smaller revisions due to the fact that it uses less weighting terms than HP filter. The amount of revisions of the HP filters are larger than the H(11) method because they have longer weighting terms and use the implied forecasts over the missing observation periods. In other words, the relatively large revision suggests that the HP filter implied model does not fit the data or forecast well. (See more detailed discussion on using more appropriate forecast models to reduce revisions in Appendix D.)

5. APPLICATIONS

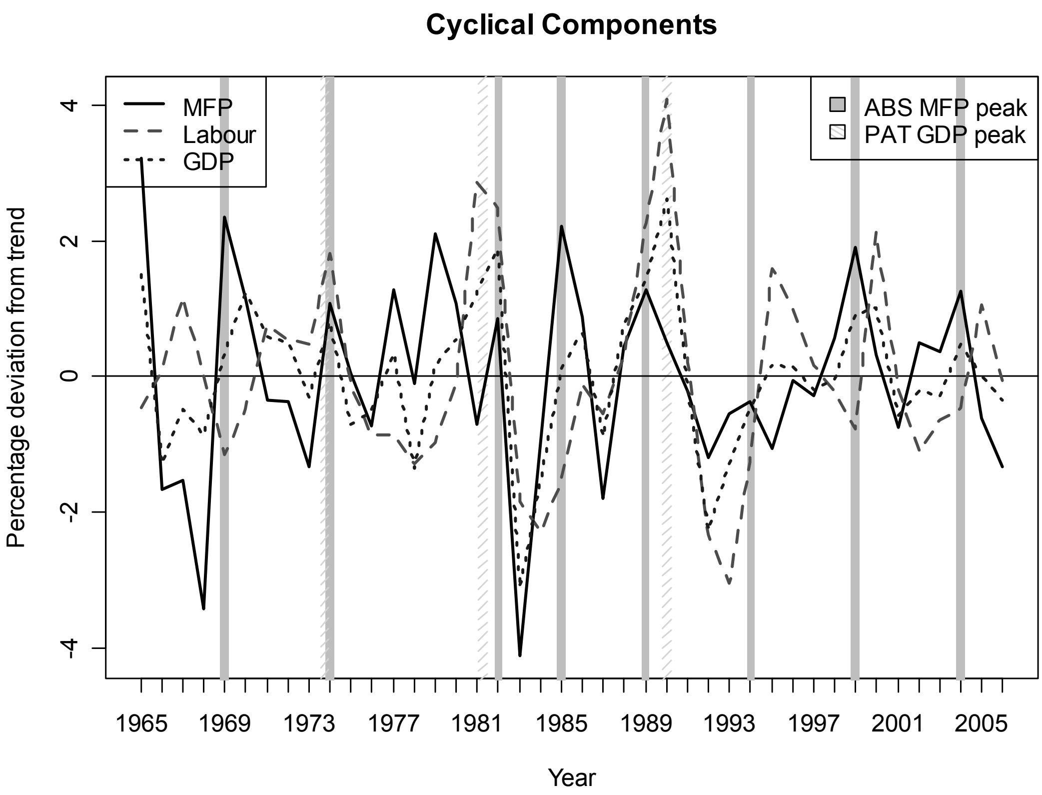
Analyses of the interactions between the cyclical behaviour of macro economic variables can provide powerful stylized facts for the general public and professional analysts alike. In this section, we present two examples of such cyclical analysis, and verify our detrending approach.

5.1 Cross-correlation between MFP, GDP and Labour inputs

The cyclical components for MFP, GDP and the labour input series were produced by using HP(25) and HP(6.25) as proposed in Section 4. Figure 5.1 shows the estimated cyclical components against the ABS declared MFP peaks, and GDP peak turning points derived from applying the well known classical Phase Average Trend (PAT) method (Boschan en Ebanks, 1978) to the quarterly GDP series.

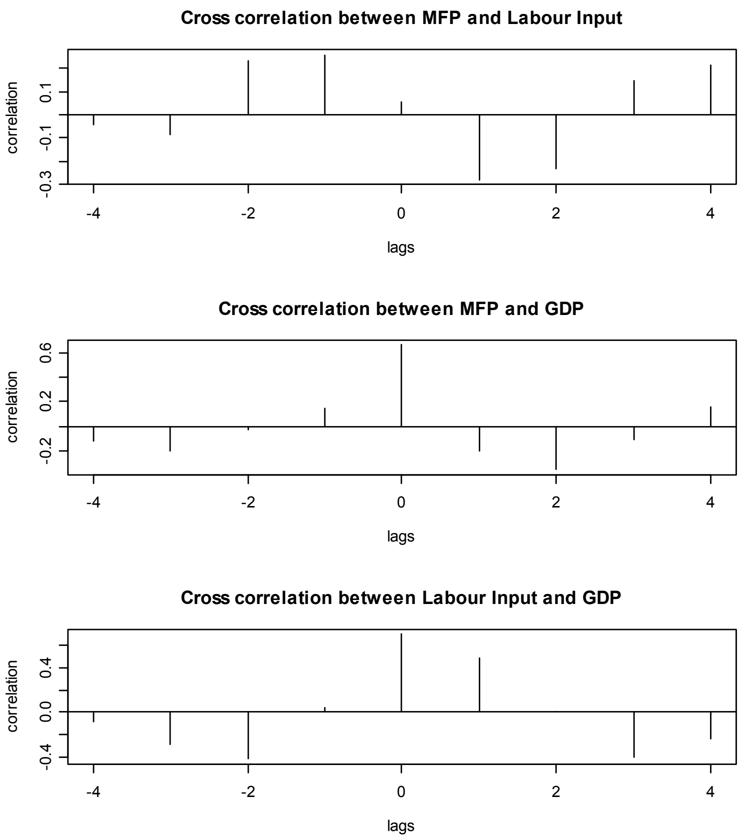
As discussed in Section 4, from a statistical evidence point of view, it appears that the 1976–77 and 1978–79 MFP peaks appear to be missing from the ABS declared MFP peaks and the 1994 peak is questionable. Our GDP peaks using annual GDP series match the peaks derived by the PAT method on quarterly GDP series. A small mismatch on 1981–82 can be attributed to the lower resolution of annual series.

5.1 Stylized cyclical components



From figure 5.1, we can observe some regular patterns among the cycles. For example, MFP peaks appear to lead GDP peaks (1968–69, 1984–85, 1987–88, 1998–99). Labour input appears to peak (or trough) at the same time as GDP or a year later (1969–70, 1973–74, 1983–84, 1989–90, 1992–93, 1999–00, 2004–05). This leading and lagging relationship can be analysed by lagged cross-correlations between cyclical components to reveal a general statistical pattern. Figure 5.2 shows the lagged cross-correlations between MFP, GDP and labour input.

5.2 Cross correlations of cyclical components



Large, positive cross correlation at lags –1 (0.26) and –2 (0.23) between MFP and labour input indicates that the MFP cycle may lead labour input by one or two years. Large negative cross correlations at lag 1 (–0.28) and 2 (–0.23) between MFP and labour input suggest that MFP cycle may move in the opposite direction to the labour input cycle with a delay of one or two years.

This leading and lagging relationship between MFP and labour input can be interpreted as labour input peaking one or two years after an MFP peak because the peaked MFP implies the work force has been over stretched and requires more labour input. A MFP trough is likely to follow a peaked labour input after one or two years because over employment may reduce productivity.

A large cross correlation between MFP and GDP at lag 0 (0.67) indicates that their cycles are likely to be synchronised or that productivity is procyclical. This stylized fact of procyclical productivity is an essential feature of business cycles (Basu and Fernald, 2000).

Large cross correlations between labour input and GDP at lag 0 (0.69) and 1 (0.49) suggest that labour input cycle is likely synchronised with GDP or has one year delay. This delay is also consistent with a well known cycle phenomenon – employment lags GDP by about half year.

5.2 Industry cycle contribution

A key analytical issue that arises with analysing MFP cycles is to understand which industries are driving the market MFP peaks at different times. This requires us to use the HP(25) filter for comparisons of cycles between different industries, noting that the use of the derived smoothing parameter (or inverse signal–noise ratio) may give inconsistent results due to the fact different smoothing parameters are applied to different industries. By applying the same filter to each series, the size of each cycle component is estimated based on the deviation from a long term trend with the same properties.

Figure 5.3 shows the cycle component of market MFP at the 2003–04 peak compared to the cycle component of the industry MFP using the HP(25) filter.

5.3 Comparing the market MFP cycle to industry MFP cycles

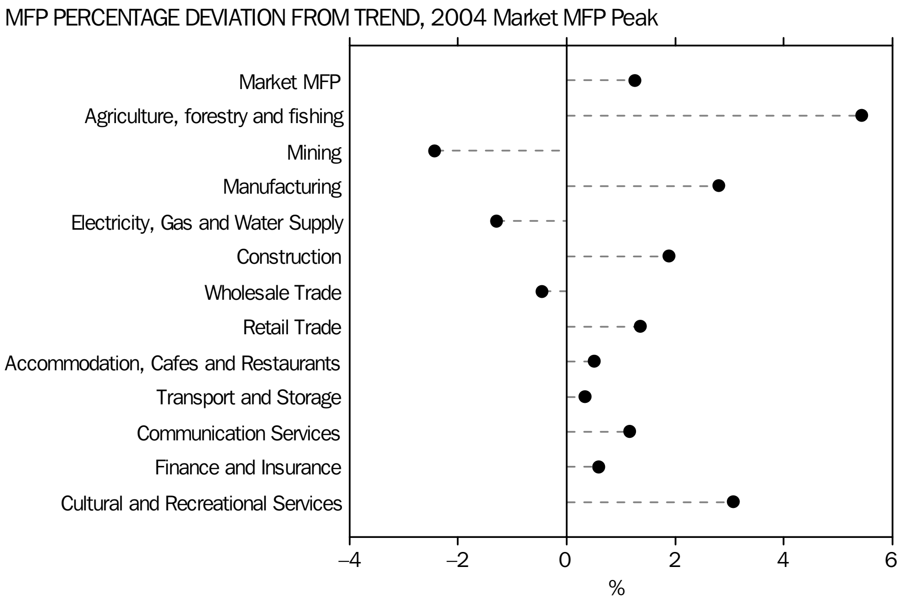


Figure 5.3 shows that in 2004, when market MFP was at the peak of its cycle, most industry cyclical components were also positive. It is not possible to get the relative contributions of the industries to the market MFP peak from just the cycle

components, as they are all rebased indexes. Hence we need to take account of the relative sizes of the different industries. To do this, we use the value added contributions measured by the proportion of output in each industry to total market output (e.g. sum of industry outputs) as a weight and apply it to the cycle component. This gives the relative contributions of the industry cycles to the total market MPF cycle peak illustrated in figure 5.4.

5.4 Weighted industry cycle contribution to market MFP cycle

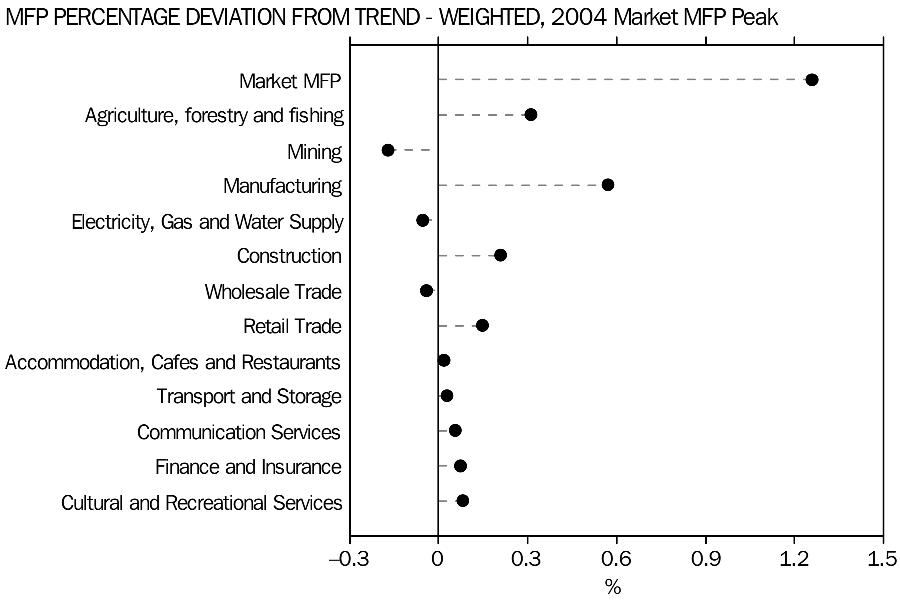


Figure 5.4 shows that the 2003–04 market MFP peak is primarily driven by Agriculture, forestry and fishing as well as Manufacturing. Construction and Retail trade also contribute somewhat to the peak and Mining is the only industry showing a strong negative effect. Similar results were observed for the other market MFP peaks with Construction behaving most similarly to market MFP and Mining showing a strong counter-cyclical effect. The reasons for the counter cyclical effect of Mining could include the lag effect, mining requires a large investment in capital over a long period of time before any output is realised.

6. CONCLUDING REMARKS

The focus of this paper was to find an appropriate statistical filter to detrend and derive the cyclical component from an ABS annual time series and compare it against current ABS practice. As a case study, we investigated the application of several commonly used filters to the annual Australian market multifactor productivity (MFP) series and evaluated their performance against the current practice of using the 11-term Henderson filter.

Our investigation suggests that the 11-term Henderson filter may not be the best choice for detrending economic series. When considering the frequency argument, the cut-off period is too short in relation to the cycle frequency range.

Using a state space unobserved component model (UCM), we estimated the signal–noise ratio which revealed the relatively high volatility of MFP. The use of a 'standard' Hodrick–Prescott (HP) filter with a smoothing parameter (or inverse signal–noise ratio) of 6.25 appeared less reliable for MFP because it did not take into account the relatively high volatility in the series, which can potentially distort the properties of the detrended series (or cyclical component).

When fitting an UCM for MFP, we found that this type of model is very sensitive to the properties of the economic series under study and it is a complicated task to find the best fitting model because of instability. Although the UCM provides very useful information and interpretation about the nature of the unobserved components and also performs good forecasts, the best fitted (or forecasting) UCM model may not necessarily meet the stationary cyclical component requirement. Therefore, a test for the existence of a stationary cyclical component is still in question. We concluded that not every UCM is suitable for cycle analysis unless the fitted model meets the predefined property/requirement of trend and cycle decomposition (see equation (1)).

In balancing the frequency argument and the level of volatility, we recommend the use of a customised Hodrick–Prescott filter with a smoothing parameter of 25 to extract the MFP cyclical component, giving consideration to comparability, consistency and transparency factors.

However, the challenge remains to verify that the MFP cyclical component derived from the HP(25) is not spurious. There is no statistical evidence suggesting the existence of a cyclical component from all random walk tests, the smooth trend and no cycle UCM. One can suspect that either the tests are not appropriate or the estimated cyclical component is spurious.

It is a well known fact that Hodrick–Prescott filter produces relatively large revisions at the end of a series because the implied model may not necessarily fit the series generation process well. In Appendix D, we have shown that the effect of revisions can be reduced by using forecast techniques. However, it would be a challenge for any official statistical organisation to define a forecast model and the forecast values for a sensitive macro time series like MFP or GDP.

The methodology presented in this paper paves the way to formalise a general ABS approach to produce long-term trend and cyclical component estimates from annual time series if there is demand for such an apporach in the future.

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APPENDIXES

A. HENDERSON FILTER AND HODRICK–PRESCOTT FILTER

The Henderson filter (Henderson, 1916) is designed to firstly, leave a time series derived from a polynomial function degree of three unchanged after this time series is filtered, and secondly, minimise the variance of the second differenced filtered series. These design features can produce a wide range of underlying curvatures with maximum smoothness. Therefore, it is commonly used as the trend-cycle component in the X–11 seasonal adjustment family of packages. The 5 and 13 term Henderson filters are generally used for quarterly and monthly time series seasonal adjustment. The ABS currently uses the 11-term Henderson filter to estimate the annual MFP trend component and derive the cyclical component. However, it has been questioned if the Henderson filter is a good choice to extract a long-term trend.

The Hodrick–Prescott (HP) filter (Hodrick and Prescott, 1997) was initially designed to estimate the trend and then derive the cyclical component of quarterly US GDP. This filter is often applied to other macro economic series for long-term trend and cyclical component estimation. This filter can be traced back as a special form of the cubic smoothing spline by Reinsch (1967) from a statistical smoothing perspective. It is also a special form of a low pass filter – the Butterworth filter family (Butterworth, 1930) from an electronic filter design perspective. Gomez (2001) and Harvey and Trimbur (2003) also suggest its application to extract trend and cycle in economic time series.

The HP filter assumes the given time series y_t of (1) and solves:

$$\min_{\{\mu_t\}}\left\{\sum_{t=1}^{T} e_t^2+\lambda\sum_{t=2}^{T-1}[(\mu_{t+1}-\mu_t)-(\mu_t-\mu_{t-1})]^2\right\} \tag{2}$$

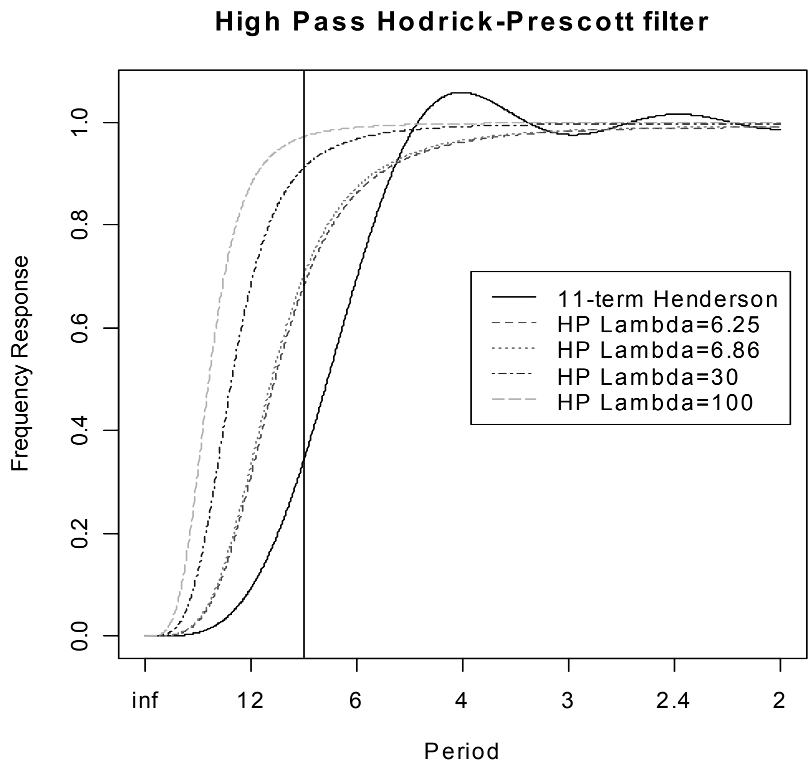
where the parameter λ controls the smoothness of the estimated trend component. Since the parameter λ is the key to control the properties of a HP filter, much has been written about the proper or correct value , however, without providing clear indications as to how to choose the appropriate value of λ.

The effect of the value of λ can be best demonstrated in the frequency domain. The HP filter gain function is given by (3):

$$g(\omega,\lambda)=\frac{4(1-\cos(\omega))^2}{1/\lambda+4(1-\cos(\omega))^2} \tag{3}$$

As the gain functions of the HP filter for different λ-values in figure A.1 show, low frequency components are allocated to the trend while high frequency components are allocated to the cycle.

A.1 Gain function of high-pass Henderson and HP filters



A.2 One-half point of HP filter with different smoothing parameters

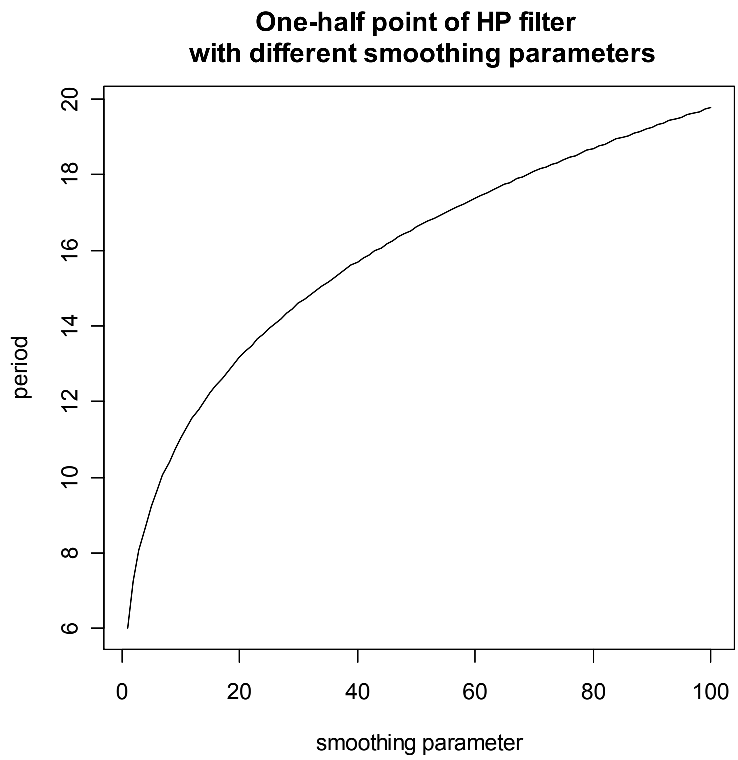


Figure A.1 suggests the 11-term Henderson filters suppress more power in low cycles (period great than eight years) than the HP filter, and amplifies the cycle in periods ranging from four to six years. In other words, the Henderson filter is likely to produce spurious cycle of 4–6 years. With HP filters, figure A.1 shows that higher values of λ shift the gain function of the trend to the left so that the trend contains less of the higher frequencies, thereby becoming smoother. If $\lambda \rightarrow \infty$, the extracted trend approaches a linear trend. With lower values of the smoothing parameter, the trend becomes more volatile as a greater proportion contains the high-frequency spectrum. In the extreme case of $\lambda = 0$, the trend is equal to the original series. Figure A.2 shows the relationship between the one-half point of the HP filter and the choice of smoothing parameter between 0.5 and 100.

Ideally, the choice of λ should be adjusted so that it reflects prior knowledge of the length of the cycle. However, the smoothing parameter does not only affect the cycle but also the volatility of the trend growth. The actual fact is that the HP filter does not contain an explicit model of the cycle. Therefore, many practitioners tend to choose high values for λ when filtering annual data because they feel that lower values, as suggested in econometric literature, would give rise to implausibly volatile trend growth rates. Thus, the value of λ is often based on a prior assumption of an acceptable trend volatility. Values of 1600 for quarterly data and of 100 for annual data are commonly used. On the basis of frequency domain considerations, Ravn and Uhlig (2002) argue that $\lambda = 1600$ for quarterly data is inconsistent with $\lambda = 100$ but would rather correspond to $\lambda = 6.25$ for annual data. Kaiser and Maravall (1999) propose a value of 8 for annual data, and Pedersen (2001) argues for a value of 1000 for quarterly data and 3.5 for annual data. Bouthevillain *et al.* (2001) used $\lambda = 30$ and Mohr (2001) used $\lambda = 20$ in annual data applications.

The frequency domain characteristics of the HP filter have well-known implications: first, the volatility of the cycle is controlled by the smoothing parameter λ. However, as λ defines the trend volatility as well, there is no way to model the trend and the cycle independently from each other. Extracting shorter cycles comes automatically at the cost of a more volatile trend. Second, the missing model for the cyclical component has important consequences when new data at the end of the sample is processed. There is no other choice than to allocate the information contained in a new data either to the trend or to the cycle, even though it may represent an outlier not generated by the data generating process underlying the HP filter. Finally, the HP filter is often used as an approximation to an ideal filter. Suppose, for instance, that the objective is to filter out a cycle length of 10 or less periods implying an ideal filter as shown in figure A.1: all frequencies below the critical frequency of $2\pi/10$ are largely cut off. By adjusting λ, the HP filter can approximate the desired ideal filter to some extent. However, there is a trade off in the choice of λ: while decreasing λ gives a better approximation to the ideal filter in the low frequency range, it worsens the approximation in the higher range. Therefore, either the trend contains frequencies which ideally should be fully captured in the cycle and is therefore overly volatile, or longer cycles which according to the ideal filter belong to the trend have too much weight in the cycle.

B. UNOBSERVED COMPONENT MODEL AND MODEL SELECTION

Harvey (1989) set up a structural state space model which explicitly contains trend, cyclical and irregular components rather than simply having an irregular term added to the trend.

$$y_t = \mu_t + c_t + \varepsilon_t, \qquad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right) \tag{4}$$

where y_t is the observed series, μ_t is the trend, c_t is the cycle, and ε_t is the irregular component.

The trend is a local linear trend defined as

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \qquad & \eta_t &\sim NID\left(0, \sigma_\eta^2\right) \\ \beta_t &= \beta_{t-1} + \zeta_t & \zeta_t &\sim NID\left(0, \sigma_\zeta^2\right) \end{aligned} \tag{5}$$

where μ_t and β_t are the trend and slope with mutually independent normal white noise disturbances η_t and ζ_t. σ_η^2 and σ_ζ^2 are their variances respectively. The stochastic cycle is generated as

$$\begin{pmatrix} c_t \\ c_t^* \end{pmatrix} = \rho \begin{pmatrix} \cos y_c & \sin y_c \\ \sin y_c & \cos y_c \end{pmatrix} \begin{pmatrix} c_{t-1} \\ c_{t-1}^* \end{pmatrix} + \begin{pmatrix} \chi_t \\ \chi_t^* \end{pmatrix} \qquad \begin{aligned} \chi_t &\sim NID\left(0, \sigma_\chi^2\right) \\ \chi_t^* &\sim NID\left(0, \sigma_\chi^2\right) \end{aligned} \tag{6}$$

where ρ is the damping factor such that $0 \le \rho \le 1$, y_c is the frequency of the cycle in radians, χ_t and χ_t^* are both $NID(0, \sigma_\chi^2)$ and independent. The irregular components $NID(0, \sigma_\varepsilon^2)$, and disturbances in all three components are taken to be independent of each other.

The local trend has a very flexible structure. If both disturbance variances σ_η^2 and σ_ζ^2 are zeros, the trend is deterministic, that is $\mu_t = \mu_0 + \beta_t$. When only σ_ζ^2 is zero, the slope is fixed and the trend reduces to a random walk with drift. Allowing σ_ζ^2 to be positive, but setting σ_η^2 to zero gives an integrated random walk trend. The signal–noise ratio is given by non-zero $\sigma_\zeta^2/\sigma_\varepsilon^2 = \lambda^{-1}$. This specification of the trend is often referred to as a smooth trend.

Harvey and Jaeger (1993) show that the HP filter is equivalent to postulating the above structural state space model imposing restrictions $\sigma_\eta^2 = 0$, $\sigma_\zeta^2/\sigma_\varepsilon^2 = \lambda^{-1}$, $c_t = 0$, $\sigma_\zeta^2/\sigma_\varepsilon^2$ is the signal–noise ratio of this restricted system with a smooth trend. The HP estimate of the cyclical component is then simply given as the smoothed irregular components. In other words, the HP filter does not actually specifically model the cyclical component but the residuals of the observed series against the estimated trend.

Harvey and others argue that it may not be appropriate to apply the HP filter to economic time series other than US GNP. Harvey (1997) states, “At the theoretical level, it can be shown to produce a spurious cycle when applied to a random walk. This is an example of the effect which Yule and Slutsky drew attention over fifty years ago. It is somewhat surprising that many modern economists still have not got the message”.

Harvey and Trimbur (2003) examine how signal–noise ratios change with the observation interval for both stock and flow time series, and look at the implications of using the Hodrick–Prescott filter to extract cycles at annual and monthly frequencies. They strongly suggest that a model building approach is the way to proceed.

Multifactor productivity

We apply the structural state space model to the annual market section multifactor productivity index. Table B.1 shows the fitted and estimated parameters.

The ‘no restriction’ model allows the model hyper-parameters to be estimated from data. The t-statistic probabilities show that the level, and slope error variances, σ_η^2 and σ_ζ^2 are not statistically different from zero. This indicates that the trend is likely to be deterministic. The cyclical component is actually non-stationary because the estimated cycle dampening factor is 1. Therefore, this model does not satisfy our predefined stationary cyclical components requirement.

The ‘no cycle’ model excludes the explicitly cyclical component. The t-statistic probability of the slope error variance σ_ζ^2 suggests that the trend is a random walk.

The ‘smooth trend–cycle’ model restricts the trend level variance σ_η^2 to zero. The t-statistic probability of the slope variance suggest a fixed slope. Therefore, the trend is likely to be a deterministic trend. This model also suggests that the cyclical component is not stationary because the estimated cyclical dampening factor is 1.

The ‘smooth trend–no cycle’ model restricts the trend level variance σ_η^2 to zero, and excludes the explicitly cyclical component. This model suggests that the estimated ‘smooth trend’ is equivalent to a HP filter with a smoothing parameter of 16.8. The cycle component is excluded in this case, indicating it may not exist.

From the parsimonious model fitting and reliable forecast perspective, the most appropriate model to describe the data generation process can be distinguished from other candidates by using information criteria. Based on the Akaike (AIC) and Bayesian (BIC) information criteria, table B.1 suggests the ‘smooth trend–cycle’ model with a non-stationary cycle component and ‘no cycle’ model are selected as the most appropriate models. Both models suggest the trend is likely to be deterministic.

Our rule for choosing the specification of the UCM model – with or without a cyclical component – is that the cyclical component will be included in the UCM model if the AIC value of the model with a cyclical component is less than the AIC value of the model without a cyclical component. We also look at whether the probability cyclical component significance test is less than 0.1, and whether the estimated cyclical component period is less than 20 years. Based on this rule, a smooth trend – without cyclical components – is chosen because the estimated cyclical period (23.56) is greater than 20 years.

In summary, the four models suggest a deterministic trend component is identified under the UCM framework. The cyclical component appears to be neither stationary nor to exist. Therefore, there is a possibility that the cyclical component derived from the HP filter may be spurious if we believe that the UCM models sufficiently fits the MFP time series.

We apply the same analysis to GDP , with results shown in table B.2 and labour input results shown in table B.3.

Table B.2 shows that the ‘smooth trend–no cycle’ model for GDP has a signal–noise ratio of 1.56, which is equivalent to a HP filter with a smoothing parameter of 0.642. This is obviously too low because the corresponding cut-off period is about 5 years. Therefore, using the standard HP(6.25) would be appropriate. Based on the AIC and BIC, table B.2 suggests the best forecasting model is the ‘no cycle’ model which describes the trend component as a random walk. The other two UCMs include stochastic cycle components. However, the significant tests on the cycle component show that the cycle component is unlikely to exist. Therefore, we believe the UCM is adequate, the results do not support a cyclical component in GDP.

Table B.3 shows that the ‘smooth trend–no cycle’ model for labour input has a signal–noise ratio of 59 which is equivalent to a HP filter with a smoothing parameter of 0.0162. This is obviously too low because the corresponding cut-off period is about four years. Therefore, using the standard HP(6.25) would be appropriate. Based on the AIC and the BIC, table B.3 suggests the best forecasting model is a ‘smooth trend – cycle’ model which describes the trend component as being close to deterministic, and a very weak cycle with a mean period of 17.2 years. However, the significant tests on the cycle component show that there is weak evidence of cycle component existence. Therefore, we believe the UCM is adequate, the results do not support a cyclical component in labour input.

B.1 Unobserved components model diagnostics for Multifactor productivity

Model fit	*No restriction*		*No cycle*		*Smooth trend – cycle*		*Smooth trend – no cycle*	
Full Log-Likelihood	97.8		94.7		97.8		92.7	
Akaike Information Criterion	–179.6		–179.3		–181.6		–177.4	
Bayesian Information Criterion	–165.7		–170.6		–169.4		–170.4	
Parameter	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*
Irregular error variance	0.000249	<.0001	0.000134	0.108	0.000249	<.0001	0.000243	0.000300
Level error variance	5.05E–14	1	0.000194	0.106	NA	NA	NA	NA
Slope error variance	2.43E–16	1	4.69E–12	0.999	2.03E–16	1	0.000015	0.235
Cycle damping factor	1.00	<.0001	NA	NA	1.00	<.0001	NA	NA
Cycle period	23.60	<.0001	NA	NA	23.60	<.0001	NA	NA
Cycle error variance	9.54E–10	0.335	NA	NA	8.99E–10	0.335	NA	NA
Significance analysis	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*
Irregular	4.52	0.034	0.41	0.524	4.52	0.034	1.00	0.318
Level	826,000	<.0001	231,000	<.0001	826,000	<.0001	173,000	<.0001
Slope	3,020	<.0001	31.20	<.0001	3,020	<.0001	0.85	0.358
Cycle	55.3	<.0001	NA	NA	55.3	<.0001	NA	NA
Signal–noise ratios	*Estimate*		*Estimate*		*Estimate*		*Estimate*	
Signal–noise ratio	2.03E–10		1.450		3.36E–13		0.0596	
Smooth parameter	4.93E+09		0.689		2.98E+12		16.8000	

B.2 Unobserved components model diagnostics for Gross domestic product

Model fit	*No restriction*		*No cycle*		*Smooth trend – cycle*		*Smooth trend – no cycle*	
Full Log-Likelihood	113.2		112.3		113.2		108.2	
Akaike Information Criterion	–210.4		–214.7		–212.4		–208.5	
Bayesian Information Criterion	–195.6		–205.4		–199.4		–201.0	
Parameter	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*
Irregular error variance	3.51E–11	0.999	3.61E–11	0.999	3.80E–11	0.999	8.37E–05	0.051
Level error variance	3.46E–09	0.999	0.000317	<.0001	NA	NA	NA	NA
Slope error variance	6.67E–07	0.784	3.78E–06	0.673	6.67E–07	0.784	0.000130	0.122
Cycle damping factor	0.93	<.0001	NA	NA	0.93	<.0001	NA	NA
Cycle period	30.80	0.022	NA	NA	30.80	0.022	NA	NA
Cycle error variance	2.52E–04	0.161	NA	NA	2.52E–04	0.161	NA	NA
Significance analysis	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*
Irregular	0	1.000	0	1.000	0	1.000	0.02	0.877
Level	208,000	<.0001	5.22E+12	<.0001	208,000	<.0001	2,790,000	<.0001
Slope	113	<.0001	30.2	<.0001	113	<.0001	3.97	0.046
Cycle	0.0	0.989	NA	NA	0.0	0.989	NA	NA
Signal–noise ratios	*Estimate*		*Estimate*		*Estimate*		*Estimate*	
Signal–noise ratio	4.69E+07		8.78E+06		0.000352		1.56	
Smooth parameter	2.13E–08		1.14E–07		2,840		0.642	

B.3 Unobserved components model diagnostics for Labour input

Model fit	No restriction		No cycle		Smooth trend – cycle		Smooth trend – no cycle	
Full Log-Likelihood	95.7		93.4		95.7		91.2	
Akaike Information Criterion	–175.4		–176.8		–177.4		–174.4	
Bayesian Information Criterion	–161.5		–168.1		–165.3		–167.5	
Parameter	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*	*Estimate*	*Pr > |t|*
Irregular error variance	1.24E–11	0.9995	2.70E–11	0.999	9.13E–12	0.9995	0.000009	0.759
Level error variance	1.31E–10	0.9996	0.000456	<.0001	NA	NA	NA	NA
Slope error variance	1.55E–12	0.9996	2.93E–11	0.999	2.93E–12	0.9995	0.0005	0.008
Cycle damping factor	0.882	<.0001	NA	NA	0.882	<.0001	NA	NA
Cycle period	17.2	0.0001	NA	NA	17.2	0.0003	NA	NA
Cycle error variance	0.000283	0.0304	NA	NA	0.000283	0.0309	NA	NA
Significance analysis	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*	*Chi-square*	*Pr > |t|*
Irregular	0	1	0	0.9999	0	1	0.02	0.892
Level	89,300	<.0001	7.85E+11	<.0001	89,300	<.0001	2,000,000	<.0001
Slope	164	<.0001	8.84	0.003	164	<.0001	0.04	0.848
Cycle	2.02	0.364	NA	NA	2.02E+00	0.364	NA	NA
Signal–noise ratios	*Estimate*		*Estimate*		*Estimate*		*Estimate*	
Signal–noise ratio	1.22E–09		1.55E+08		2.29E–09		59.0	
Smooth parameter	8.22E+08		6.45E–09		4.36E+08		0.02	

C. A FLAW IN THE FREQUENCY ARGUMENT AND A MODEL-BASED REMEDY

The frequency argument states that an appropriate filter should be able to separate the long-term trend and stationary (including cycle) components from a time series with predetermined periods. However, a detrending filter will also cut out the low frequency power of the stationary component. In other words, this low frequency power overflows into the low frequency range as a part of the trend. Therefore, a detrending filter can potentially distort the properties of the detrended series if care has not to be taken on the proportion of low frequency power in the trend and stationary components. However, this proportion can be estimated using the signal–noise ratio in the UCM model.

We conducted a simple simulation to illustrate the flaw of the frequency argument by generating a stochastic trend from an integrated random walk model.

$$(1-B)^2 y_t = \varepsilon_t, \qquad \varepsilon_t \sim NID(0,1) \tag{7}$$

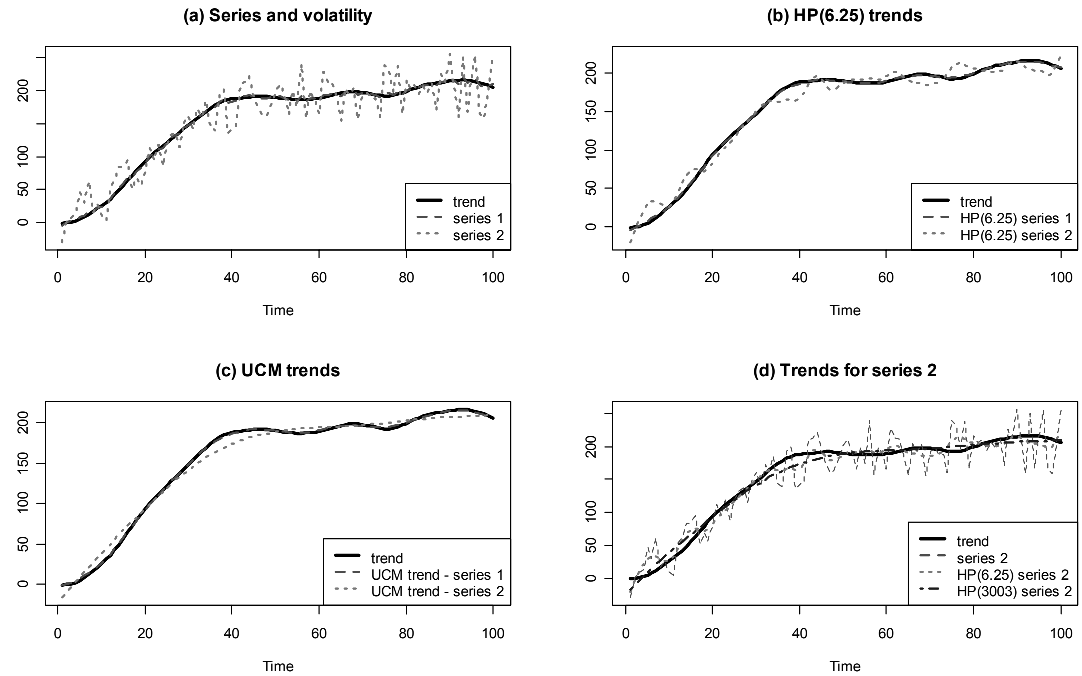
A ‘benchmark’ smooth trend, u_t, was derived from applying HP(6.25). We then generate dtwo time series by adding the same white noise but with different standard deviations (of 2.5 and 25 respectively).

$$\begin{aligned} &y_{i,t} = \mu_t + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} = c_i \varepsilon_t \quad i = 1,2 \\ &\text{where } c_1 = 2.5 \text{ and } c_2 = 25 \end{aligned} \tag{8}$$

By construction, these two series are co-integrated and share the same trend. We next applied a HP(6.25) filter, a UCM model and a HP filter with the smoothing parameter estimated by the UCM model as the inverse value of the signal–noise ratio.

Figure C.1 below shows one realisation of a sample of 100. Graph (a) plots the ‘benchmark’ trend and the two series generated with standard deviations of 2.5 and 25 respectively. Graph (b) shows that for series 1, the HP(6.25) trend is almost the same as the ‘benchmark’ trend while for series 2, the HP(6.25) trend is not as smooth and is not a good approximation of the ‘benchmark’ trend. Graph (c) shows that with series 2, the UCM trend has a better approximation and is smoother than the series 2 HP(6.25) trend. Graph (d) plots the ‘benchmark’ trend, series 2, series 2 HP(6.25) trend, and HP(3003) trend where the smoothing parameter is derived from the UCM model signal–noise ratio estimate. From this example, we can see that a predetermined smoothing parameter of 6.25 is not suitable to extract the trend when a larger volatility of series 2 is observed.

C.1 Simulation results

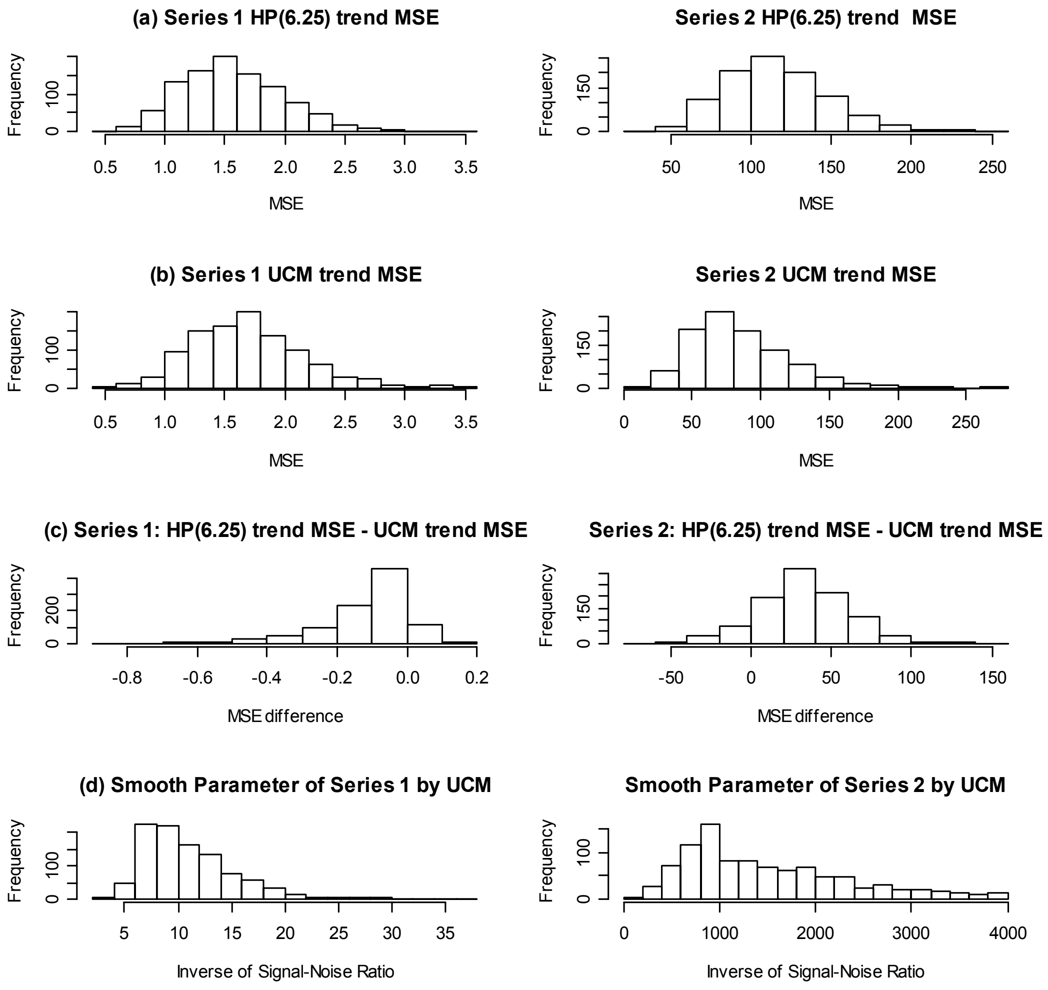


We conducted 1000 empirical simulations (note: 1000 replicates simulations are selected when UCM model fittings converge, the convergence rate was about 85%) as above. The closeness of the estimate trends to the corresponding 'benchmark' in each simulation was measured by the mean square error (MSE).

Figure C.2 shows the empirical distribution of MSEs. Comparing graphs in rows (a) and (b), we can see that with series 1, the HP(6.25) trend and the UCM trend performed equally well. However, the MSE distribution of series 2 showed that the HP(6.25) trend centres at 100~125 with a median of 113 while the MSE distribution of the series 2 UCM trend centres at 65~80 with median of 78. In other words, with series 2, the UCM trend is likely to be much closer to the 'benchmark' trend in each simulation. A pair of graphs in row (c) show the empirical distribution of differences between the MSE of the HP(6.25) and the MSE of the UCM trend for each sample simulation.

Flat likelihood functions and numerical optimisation induced parameter estimation errors may contribute to a worsening performance of the UCM trend. For series 2, however, the HP(6.25) trend MSE is greater than the corresponding UCM trend MSE, and their differences have a median of 31. A pair of graphs in row (d) show the empirical distributions of smooth parameters derived from the UCM signal–noise ratio for the two series. For series 1, the empirical smoothing parameter distribution is skewed to right, has a peak at about 8 with median 10. Because the estimated smoothing parameters are not very different from the predetermined smoothing parameter 6.25, HP(6.25) performs well. For series 2, the empirical smoothing parameter distribution is also skewed to right, but has a peak at about 900 with median 1270. The estimated smoothing parameters are different from the predetermined smoothing parameter (6.25). This indicates that the domination of the spectrum power of the stationary component at low frequencies range can largely distort trend estimate by a predetermined HP(6.25). A remedy to handle this situation is to use a HP filter with a customised smoothing parameter derived from the signal–noise ratio estimated from a suitable UCM.

C.2 MSEs from 1000 simulations



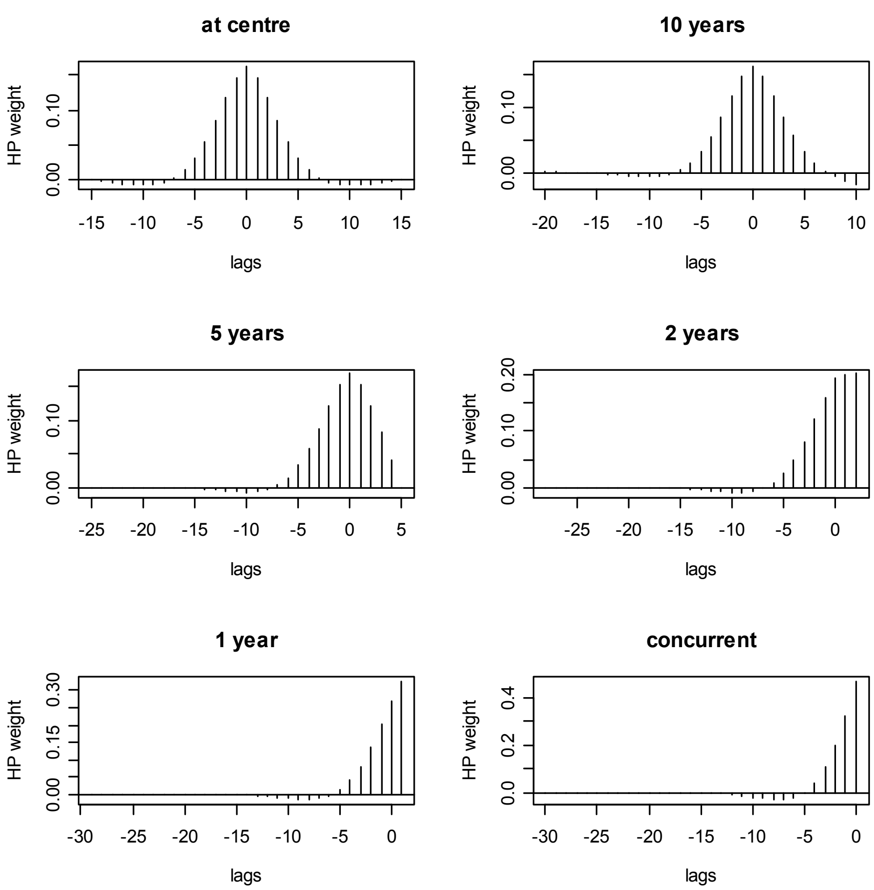
From this simulation study, we can confidently conclude that a predetermined smoothing parameter HP filter with a desirable frequency cut-off should be only used as a low boundary. For example, 6.25 should be used as the minimum HP filter smoothing parameter if the cut-off is set at about ten years. A smooth trend UCM fitting excise can provide guidance on the choice of an appropriate smoothing parameter, which takes care of the possible high volatility of a time series, and estimates the trend component in a more accurate way if the UCM fitting is satisfactory.

D. THE ENDPOINT PROBLEM AND REVISIONS

Many trend-cycle decomposition methods suffer from the so called endpoint problem. The trend estimate at t is based on information available up to and including period t. It can change significantly if new data for period $t+1$ become available irrespective of the value of the new data point. Near the end of the sample, less information is available regarding the persistence of shocks, meaning less reliable decomposition of the trend and cycle components.

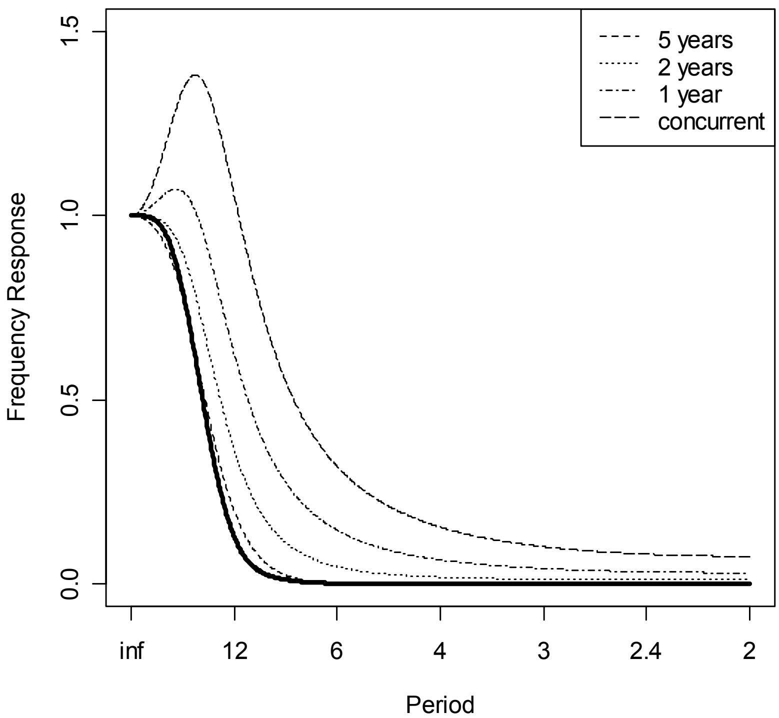
The real-time allocation of the dynamics to trend and cyclical forces is uncertain as information is missing. It is only when new data in future periods become available that the trend-cycle decomposition in period t becomes more certain and stabilises. While the limited amount of real-time information is a general problem for any trend cycle decomposition, using a two-sided symmetric filter that relies on past and future periods trend extraction methods differ in the significance of the problem. The problem is less significant when the model underlying the filter can forecast the original time series well. An endpoint problem exists only if the stochastic model underlying the filter is a weak representation of the data generating process.

D.1 HP(25) filter weights



Kaiser and Maravall (1999) show that the HP filter is consistent with its own forecast which are embedded in the end-point weighting pattern. Figure D.1 and D.2 show the HP(25) weighting pattern in the time domain and gain functions with different end-points, respectively. It is clear to see that the concurrent end-point gain function is very different from the centre filter and the differences diminish as more data points become available. In other words, the concurrent estimates are the most unreliable, and estimates after two periods are acceptable. The HP filter implied model is not very comprehensive – assuming that the stationary part is white noise, the second order random walk property of the trend is the only prior piece of information that can be exploited for forecasting. Furthermore, the HP filter provides practically no means to adjust it to the data. Hence, its forecast performance cannot be improved.

D.2 Gain function of 'asymmetric' HP(25) filter

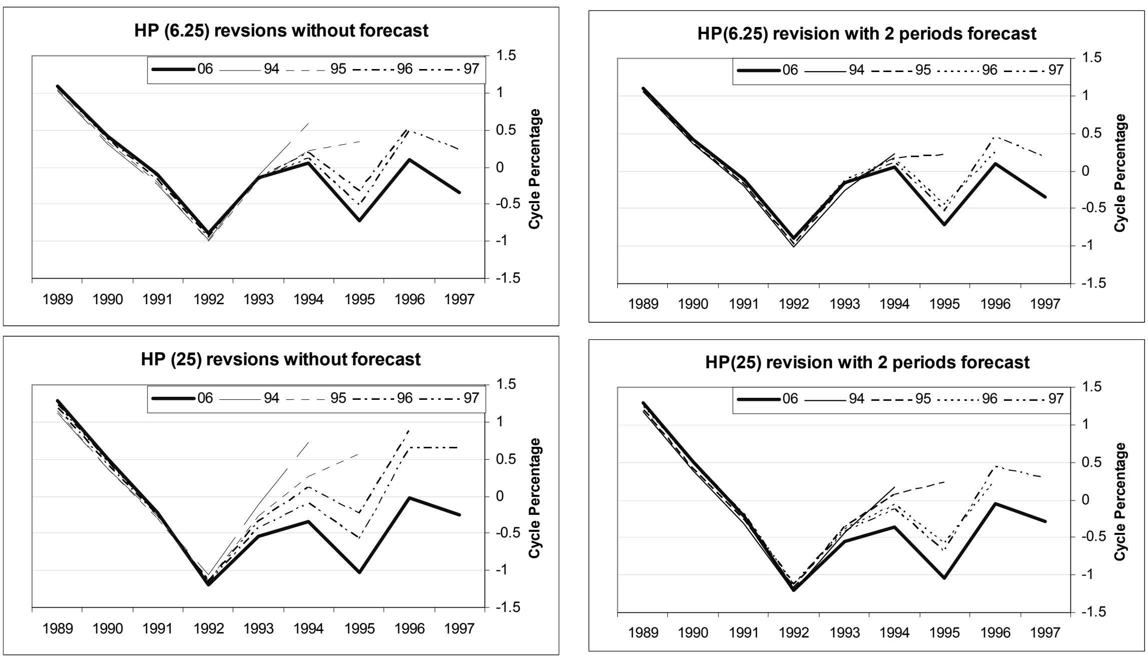


As a standard remedy to the end-point problem, the economic time series is extended by forecasts, and the filter is applied to the extended series. If the forecast turns out correct ex post, then there will be no end-point bias. However, this approach comes with other problems. It is unclear how the filter processes forecast errors, which may translate into errors in the trend estimation. Even if the forecast itself is unbiased and the forecast error is a random white noise process, it is unlikely that the errors computed as part of the trend share this feature because the filter model will differ from the underlying forecast model.

If there are good reasons to assume that there exists a model with better forecasting performance than the filtering model, the former should be applied for the trend-cycle decomposition. This will improve the filter model's consistency with the data generating process. It follows that the end-point problem should be alleviated by improving the forecast performance of the stochastic filter model, i.e. fitting the actual data. However, although a forecast model may have good forecast performance, it is not always true that this model can provide a predefined property of trend-cycle decomposition. For example, a forecast model can have a non-stationary cycle which does not satisfy the precondition that the cyclical component must be stationary.

We use the UCM model as forecast model for the total MFP series to illustrate the relationship between revisions and forecasts. As mentioned in the last section, the smooth trend – cycle model specification is selected as the best model among other candidate model specifications based on AIC criterion. We use this model specification to forecast a two period leading series and then apply the HP filter to produce cyclical component estimates. Figure D.3 shows the revisions of cyclical component estimates for 1994 using HP(6.25) and HP(25) with and without forecasts. The latest estimates are presented in the bold curve based on the up to 2006.

D.3 Revision of cyclical component estimates of 1994



We can see clearly that the first (or concurrent) estimates are not reliable for both HP(6.25) and HP(25) without forecast. With two period forecasts, the estimates are improved for both HP(6.25) and HP(25). From these revision graphs we draw the following general conclusions:

- The revisions induced by the end-point problem can be reduced by using appropriate forecasts
- The larger the HP smoothing parameter, the larger revision size and the longer lasting the revision.

The preceding discussion highlights some important points. It is difficult to give a universal ranking of the statistical methods. Section 4 shows that most methods provide estimates with a similar overall profile of potential turning points but there are large divergences on the assessment of the magnitude of the estimated cycles. The ‘consistency with priors’ and the ‘difference between real-time and final estimates’ (or revisions) are important criteria to select an appropriate filter in practice. Whatever filter is used, it is important to bear in mind its underlying assumptions and its shortcomings, as well as to make a critical and non mechanical use of it.

E. INDUSTRY LEVEL ANALYSIS

Testing of the industry level MFP estimates was undertaken and compared to the results for market MFP, GDP and labour input. A wide range of signal–noise ratios were obtained, shown in table E.1, using the UCM Kalman filter. [5] The unit root test suggests that all industry level MFP series cannot be decomposed into a deterministic time trend plus stationary series. There is no strong evidence from the Box–Ljung Q-test or the Durbin–Watson test against the hypothesis that they are simply random walks. A large range of inverse signal–noise ratios is observed. This suggests that the industry level MFPs have a large range of volatility levels. It is not appropriate to apply the 'standard' HP(6.25) to extract cyclical components for Agriculture, forestry and fishing; Construction; Communication services; and Cultural and recreational services because they are identified to have large inverse signal–noise ratios when the UCM without stochastic component model is applied. The UCM with stochastic component specification identifies that Mining; Manufacturing; Construction; Accommodation, cafes and restaurants; and Communication service industries are likely to have stochastic cyclical components.

5 The signal–noise ratios are estimated from the UCM Kalman filter method with a specification of smooth trend with and without stochastic cycle component. See details in Appendix B.

E.1 Signal–noise ratio estimated by UCM

	I(1) with deterministic time trend	*Random walk*		*UCM doesn't include stochastic cycle*	*UCM includes stochastic cycle*	*Final UCM model*
	Dickey–Fuller unit root tests	*Box–Ljung Q(6) p-value*	*Durbin–Watson p-value*	*Inverse of signal–noise ratio (lambda)*	*Inverse of signal–noise ratio (lambda)*	*Inclusion of stochastic cycle component*
Market MFP	Yes	0.336	0.242	16.8	2.98E+12	No
Total GDP	Yes	0.553	0.002	0.6	2,840	No
Labour Input – Hours Worked	No	0.082	0.342	0.0	4.36E+08	Yes
Agriculture, forestry and fishing	Yes	0.133	0.715	273.4	273.0	No
Mining	Yes	0.994	0.446	2.6	6.4	Yes
Manufacturing	Yes	0.836	0.751	0.9	293.0	Yes
Electricity, Gas and water supply	Yes	0.000	0.570	0.2	1.0	No
Construction	Yes	0.361	0.823	34.2	20.7	Yes
Wholesale trade	Yes	0.301	0.104	0.2	4.01E+07	No
Retail trade	Yes	0.950	0.685	0.4	1.96E+08	No
Accommodation, cafes and restaurants	Yes	0.068	0.783	1.8	1.8	Yes
Transport and storage	Yes	0.962	0.804	0.5	2.77E+08	No
Communication services	Yes	0.071	0.804	13.5	10.8	Yes
Finance and insurance	Yes	0.892	0.592	2.0	27.9	No
Cultural and recreational services	Yes	0.439	0.278	15.2	33.5	No

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